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RECONSTRUCTION OF TURBULENCE SPECTRUM FROM TRANSIENT CHARACTERISTICS OF A SHADOW-INSTRUMENT SIGNAL

Yu. I. Kopilevich UDC 532.507

With the investigation of turbulence using a shadow instrument with photoelectric recording, the statistical characteristics of the signal taken off from the instrument are used to obtain information on the statistics of the investigated medium [1, 2]. In situations where the investigated medium is moving perpendicular to the instrument axis (for example, with experiments in hydro- and aerodynamic tubes), it is convenient to use the transient characteristics of the signal. In the present article an investigation is made of the connection of the transient correlation function and the frequency spectrum of a shadow-instrument signal with the energy spectrum of the optical inhomogeneities in the medium; a method is given for reconstructing the spectrum of the inhomogeneities from the correlation function or the transient spectrum of the signal.

§1. Connection between the Correlation Function of the Signal and the Fourth Moment of the Light Field

The general scheme of the shadow instrument is given in Fig. 1. A coherent monochromatic light beam from the illuminator 1 passes through a layer of the investigated medium with thickness L, situated between the planes 2 and 3, and is reflected by the lens 4 on its focal plane 5. In the plane 5 (the shadow plane) there is a shadow diaphragm; the light passing through the shadow plane is collected by the lens 6 and sent to the photomultiplier 7. In what follows, by the "instrument signal" we shall understand the intensity of the light falling on the photomultiplier (and not the photomultiplier current).

We introduce the Cartesian coordinates x, y, z in such a way that the z axis will be directed along the axis of the light propagation; plane 2 corresponds to $z = 0$, plane 3 to $z = L$. Let $u(x, y, L, t) \equiv u(\mathbf{x}, t)$, $\mathbf{x} = (x, y)$ be the random distribution of the field at the plane 3 at the moment of time t. Then the instantaneous value of the signal of the instrument $E(t)$ is [3]

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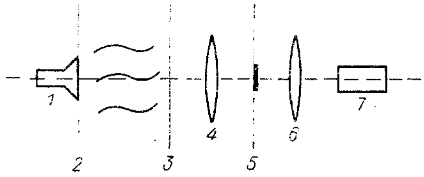

Fig. 1

$$E(t)=\frac{1}{(2\pi)^2}\int d\mathbf{x}_1\int d\mathbf{x}_2\int d\varkappa u(\mathbf{x}_1,t)\bar{u}(\mathbf{x}_2,t)e^{i\varkappa(\mathbf{x}_1-\mathbf{x}_2)}\chi(\varkappa), \tag{1.1}$$

where the coordinates $\varkappa = (\varkappa_1, \varkappa_2)$ in the plane 5 are connected with the coordinates $\mathbf{x} = (x, y)$ by the relationship $\varkappa = (2\pi/\lambda f)\mathbf{x}$ (λ is the wavelength of the light; f is the focal distance of the lens); $\chi(\varkappa)$ is the transmission function of the shadow diaphragm with respect to the intensity. For the correlation function of the deviations in the value of the instrument signal from its mean value

$$K(t_1,t_2)\equiv\langle(E(t_1)-\langle E(t_1)\rangle)(E(t_2)-\langle E(t_2)\rangle)\rangle$$

(the angular brackets denote averaging with respect to the ensemble of the realizations of a random medium), from (1.1) we obtain

$$K(t_1,t_2)=\frac{1}{(2\pi)^4}\int d\mathbf{x}_1\int d\mathbf{x}_2\int d\mathbf{x}_3\int d\mathbf{x}_4\int d\varkappa_1\int d\varkappa_2\chi(\varkappa_1)\chi(\varkappa_2)\times \\ \times e^{i\varkappa_1(\mathbf{x}_1-\mathbf{x}_2)}e^{i\varkappa_2(\mathbf{x}_3-\mathbf{x}_4)}\hat{\Gamma}(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3,\mathbf{x}_4;t_1,t_2), \tag{1.2}$$

where $\hat{\Gamma}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4; t_1, t_2) \equiv \Gamma(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4; t_1, t_2) - \Gamma(\mathbf{x}_1, \mathbf{x}_2; t_1)\Gamma(\mathbf{x}_3, \mathbf{x}_4; t_2)$; $\Gamma(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4; t_1, t_2) \equiv \langle u(\mathbf{x}_1, t_1)\bar{u}(\mathbf{x}_2, t_1)u(\mathbf{x}_3, t_2)\bar{u}(\mathbf{x}_4, t_2)\rangle$ is the fourth two-time moment of the field u in the plane z = L; $\Gamma(\mathbf{x}_m, \mathbf{x}_n; t) \equiv \langle u(\mathbf{x}_m, t)\bar{u}(\mathbf{x}_n, t)\rangle$ is the second single-time moment. The quantity $\hat{\Gamma}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4; t_1, t_2)$ will be called the centered fourth two-time moment of the field at the plane z = L.

The random field of the optical inhomogeneities in the medium will be assumed stationary; in this case, the single-time moments do not depend on the time, while the two-time moments depend only on the modulus of the difference of their time arguments

$$\Gamma(\mathbf{x}_1,\mathbf{x}_2;t_1)\equiv\Gamma(\mathbf{x}_1,\mathbf{x}_2),\ \Gamma(\mathbf{x}_3,\mathbf{x}_4;t_2)\equiv\Gamma(\mathbf{x}_3,\mathbf{x}_4)$$
$$\Gamma(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3,\mathbf{x}_4;t_1,t_2)\equiv\Gamma(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3,\mathbf{x}_4;\tau),$$
$$\hat{\Gamma}(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3,\mathbf{x}_4;t_1,t_2)\equiv\hat{\Gamma}(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3,\mathbf{x}_4;\tau),\ K(t_1,t_2)\equiv K(\tau),$$

where $\tau = |t_1 - t_2|$.

§2. Calculation of the Centered Fourth Two-Time Moment of the Light Field

We use the following postulation with respect to the random medium. The field of the dielectric permeability $\varepsilon(\mathbf{r}, t)$, $\mathbf{r} = (x, y, z)$ will be assumed to be stationary, statistically homogeneous, and isotropic. The fluctuations of the dielectric permeability are assumed to be small

$$\varepsilon(\mathbf{r},t)=\langle\varepsilon\rangle(1+\varepsilon'(\mathbf{r},t)),\ |\varepsilon'(\mathbf{r},t)|\ll 1,$$

$\langle\varepsilon\rangle$ is the mean value of the dielectric permeability, which, by virtue of the assumptions adopted, does not depend on the coordinates or the time.

The field $u_0(\mathbf{x})$ in the plane 2 is given in the form

$$u_0(\mathbf{x})=A\exp\{-\mathbf{x}^2/2a^2\},$$

where a is the effective radius of the beam; A is the amplitude at the axis of the beam.

We shall also use the usual assumptions

$$l\gg\lambda,\ a\gg\lambda,\ l\ll L,$$

where l is the dimension of the smallest homogeneities in the medium. We use the results obtained in [4]. It is evident that the expressions of interest to us for the centered fourth two-time moment of the light field in the plane 3 can be obtained from the formula for the centered fourth single-time moment [4] by replacing the two-dimensional spectrum of the turbulence $\Phi(\eta)$ by a two-dimensional Fourier transform $\Phi_1(\eta;\ \tau)$ of the transient correlation function of the fluctuations of the dielectrical permeability

$$\sigma_1(\mathbf{r};\tau) \equiv \langle \varepsilon'(\rho + \mathbf{r},\ t+\tau)\varepsilon'(\rho,\ t)\rangle:$$

$$\Phi_1(\eta;\tau) = \frac{1}{(2\pi)^2}\iiint\limits_{-\infty}^{\infty} \sigma_1(\mathbf{r};\tau)\, e^{i(\eta_x x+\eta_y y)} dx dy dz. \tag{2.1}$$

Thus, we obtain

$$\widehat{\Gamma}(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3,\mathbf{x}_4;\tau) = \Gamma_{1001} + \Gamma_{0110} + \Gamma_{1010} + \Gamma_{0101} + O(\varepsilon'^3), \tag{2.2}$$

$$\Gamma_{lmnp} \equiv \langle V_l(\mathbf{x}_1,t)\overline{V}_m(\mathbf{x}_2,t)V_n(\mathbf{x}_3,t+\tau)\overline{V}_p(\mathbf{x}_4,t+\tau)\rangle,$$

where $V_i(\mathbf{x},\ t)$ is the i-th term of the Born expansion of the field $u(\mathbf{x},\ t)$ in the plane 3 [5];

$$\Gamma_{1001} = \Gamma_{0000}\frac{k^2}{4}\int\limits_0^L dp \int d\eta \Phi_1(\eta;\tau) \times$$

$$\times \exp\left\{-\eta^2 \frac{(L-p)^2}{k^2a^2|B(L)|^2} + i\beta(p)\mathbf{x}_1\cdot\eta - i\overline{\beta}(p)\mathbf{x}_4\cdot\eta\right\}; \tag{2.3}$$

$$\Gamma_{1010} = -\Gamma_{0000}\frac{k^2}{4}\int\limits_0^L dp \int d\eta \Phi_1(\eta;\tau) \times$$

$$\times \exp\left\{-\eta^2\frac{(L-p)^2}{k^2a^2|B(L)|^2} - i\eta^2\left(1+\frac{pL}{k^2a^4}\right)\frac{L-p}{k|B(L)|^2} + \right. \tag{2.4}$$

$$\left. + i\beta(p)\mathbf{x}_1\cdot\eta - i\beta(p)\mathbf{x}_3\cdot\eta\right\},$$

Γ_{0110} and Γ_{0101} are obtained from Γ_{1001} and Γ_{1010}, respectively, by an operation of complex conjugation and permutation of $\mathbf{x}_1$ with $\mathbf{x}_2$ and $\mathbf{x}_3$ with $\mathbf{x}_4$. In (2.3), (2.4), the notation of [4] is retained:

$$B(z) \equiv 1 + iz/ka^2,\quad \beta(p) \equiv B(p)/B(L).$$

§3. Correlation Function and Frequency Spectrum of Shadow Instrument

Substituting (2.3), (2.4), into (2.2), we obtain the fourth moment $\hat{\Gamma}(\mathbf{x}_1,\ \mathbf{x}_2,\ \mathbf{x}_3,\ \mathbf{x}_4;\ \tau)$. Now, from (1.2) we have

$$K(\tau) = k^2A^4a^8\int d\eta\Phi_1(\eta;\tau)P(\eta), \tag{3.1}$$

where

$$P(\eta) = \frac{1}{4}e^{-a^2\eta^2}\int\limits_0^L dp\,|\Psi(\eta,p) - \overline{\Psi}(-\eta,p)|^2,$$

$$\Psi(\eta,p) = e^{-i\frac{p\eta^2}{2k}}\int \chi(\varkappa)\, e^{-a^2\varkappa^2}\exp\left\{-a^2\left(1+i\frac{p}{ka^2}\right)\eta\varkappa\right\}d\varkappa. \tag{3.2}$$

We find the connection between $\Phi_1(\eta;\ \tau)$ and the two-dimensional spectrum of the turbulence $\Phi(\eta) \equiv \Phi_1(\eta;\ 0)$ adopting the hypothesis of "frozen turbulence" [6]. In this case

$$\sigma_1(\mathbf{r};\tau) \equiv \langle\varepsilon'(\rho+\mathbf{r},t+\tau)\varepsilon'(\rho,t)\rangle = \langle\varepsilon'(\rho+\mathbf{r}-\mathbf{v}\tau,t)\varepsilon'(\rho,t)\rangle \equiv \sigma_1(\mathbf{r}-\mathbf{v}\tau;0)$$

and from (2.1) it follows

$$\Phi_1(\eta;\tau) = e^{i\eta\mathbf{v}\tau}\Phi(\eta), \tag{3.3}$$

where the orthogonality of $\mathbf{v}$ with respect to the z axis is used. Substituting (3.3) into (3.1), we obtain

$$K(\tau) = k^2A^4a^8\int\Phi(\eta)P(\eta)e^{i\eta\mathbf{v}\tau}d\eta. \tag{3.4}$$

In what follows we shall limit ourselves to the case where the transition function of the shadow diaphragm $\chi(\varkappa)$ depends only on the modulus of $\varkappa$: $\chi(\boldsymbol{\varkappa}) \equiv \chi(\varkappa)$. Under these circumstances, from (3.2) it follows that the function $P(\eta)$ is also independent of the angle:

$P(\eta) \equiv P(\eta)$. By virtue of the assumption of the isotropic character of the field of the inhomogeneities, the spectrum of Φ is a function only of the modulus of η, i.e., $\Phi(\eta) \equiv \Phi(\eta)$, and, in (3.4), we can integrate over the angles. Thus, we obtain

$$K(\tau) = 2\pi k^2 A^4 a^8 \int_0^\infty \Phi(\eta) P(\eta) J_0(\eta v \tau) \eta d\eta, \tag{3.5}$$

where $J_0(z)$ is a Bessel function of zero-th order.

Expression (3.5) is a Hankel transform of zero-th order [7] of the function $2\pi k^2 A^4 a^8 \cdot \Phi(\eta) P(\eta)$. We note that the two-dimensional spectrum of $\Phi(\eta)$ is connected with the three-dimensional Fourier transform of the correlation function $F(\eta)$ by the relationship $\Phi(\eta) = 2\pi F(\eta)$.

For the frequency spectrum $S(\nu)$ of the shadow instrument signal

$$S(\nu) \equiv \frac{1}{2\pi} \int_{-\infty}^{\infty} K(\tau) e^{i\nu\tau} d\tau$$

from (3.5) there follows the expression

$$S(\nu) = \frac{2k^2 A^4 a^8}{v} \int_{\nu/v}^{\infty} \Phi(\eta) P(\eta) \frac{\eta}{\sqrt{\eta^2 - \frac{\nu^2}{v^2}}} d\eta, \tag{3.6}$$

which is an Abel [7] transformation of the function $(2k^2 A^4 a^8 / v)\Phi(\eta)P(\eta)$. For the integral transformations (3.5), (3.6), there exist inversion formulas

$$\Phi(\eta) = v^2 [2\pi k^2 A^4 a^8 P(\eta)]^{-1} \int_0^\infty K(\tau) J_0(\eta v \tau) \tau d\tau; \tag{3.7}$$

$$\Phi(\eta) = - v^2 [\pi k^2 A^4 a^8 P(\eta)]^{-1} \int_{\eta v}^{\infty} \frac{S'(\nu)}{\sqrt{\nu^2 - \eta^2 v^2}} d\nu. \tag{3.8}$$

Expressions (3.5)-(3.8) give a formal solution to the problem of the reconstitution of the spectrum of the turbulence; however, for an actual solution of the problem, the use of one or another method of regularization is required [8, 9].

We note that in the problem of reconstitution the apparatus function $P(\eta)$ plays a considerable role. As an example, let us consider the case of a "Gaussian" shadow diaphragm

$$\chi(\varkappa) = 1 - e^{-a^2\varkappa^2}.$$

The function $P(\eta)$ in (3.5), (3.6) for such a diaphragm, with satisfaction of the conditions

$$(L\lambda/l^2)^2 \ll 1, \; (L\lambda/al)^2 \ll 1$$

is equal to

$$P(\eta) = \frac{\pi^3}{96} \frac{L^3}{k^2 a^4} \exp\left(-\frac{3}{4} a^2 \eta^2\right) \eta^4.$$

The weighting function $P(\eta)\eta$ is maximum with $\eta_* = (\sqrt{10/3})a^{-1} \simeq 1.8a^{-1}$, whose acuteness can be judged from the "half-width"

$$\Delta \equiv \frac{\int_0^\infty \exp\left(-\frac{3}{4} a^2 \eta^2\right) \eta^5 d\eta}{\exp\left(-\frac{3}{4} a^2 \eta_*^2\right) \eta_*^5} \simeq 1{,}4\, a^{-1}.$$

It is clear that the part of the spectrum of the inhomogeneities lying outside the interval $(\eta_* - \Delta, \eta_* + \Delta)$ has no active effect on the functions $K(\tau)$ and $S(\nu)$; therefore, the reconstitution of this part of the spectrum is impossible. Consequently, the shadow instrument parameters (the effective radius of the beam) must be selected depending on the region of spectral numbers in which it is desired to establish the energy spectrum of the turbulence.

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EXPERIMENTAL STUDY OF DYNAMICS OF GAS BUBBLES IN A TURBULENT JET

A. I. Mart'yanov

UDC 534.833.53

In investigating the scattering of ultrasound by turbulent jets for noncontacting flow diagnostics, the effect of gas bubbles of various sizes must be taken into account. The problem of the evolution of the bubble distribution function in the jet is also of certain interest.

A number of experimental papers have appeared on the study of the free-gas content in still water and in a disturbed volume of liquid. Gavrilov [1] describes a method for determining the free-gas content based on a measurement of the attenuation of ultrasound.

In this method the gas content is estimated from the expression

$$K_l = 6{,}3 \cdot 10^5 \, n\,(R_0) R_0^3,$$

where K_l is the attenuation factor in dB/m; $n(R_0)$ is the number of bubbles per cm³ of liquid; and R_0 is the radius of a bubble.

We find the gas-bubble distribution function by using the tabulated values [2] of the absorption cross section σ for bubbles of various sizes. Since the composition of the gas in the bubbles is uncertain, the actual and calculated values of the absorption cross section differ somewhat. Nevertheless, a knowledge of the frequency dependence of σ permits a study of the variation of the bubble-distribution function along the jet.

The intensity of an ultrasound wave propagating in a medium containing bubbles varies according to the law [3]

$$W(x) = W_0 \epsilon^{-n_R \sigma_R R^3}, \tag{1}$$

where $W(x)$ is the wave intensity after penetrating a distance x into the layer with bubbles; W_0 is the wave intensity at the entrance to the layer; n_R is the number density of bubbles of radius R; and σ_R is the absorption cross section of a bubble of radius R.

It is well known [3] that the absorption of sound energy by a gas bubble is maximum at a frequency equal to the resonance frequency of the bubble [4]

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